assessment

Power Calculation

Learn to solve this type of problems, not just this problem!

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(1) Compute the sums of the following expressions:

i) $1 \times 2 \times 3 + 2 \times 3 \times 4 + \cdots + 2016 \times 2017 \times 2018$

ii) $\frac{1}{1\times2\times3} + \frac{1}{2\times3\times4} + \cdots + \frac{1}{2016\times2017\times2018}$

(2) Let

$$f(r) = \sum_{j=2}^{2016} \frac{1}{j^r} = \frac{1}{2^r} + \frac{1}{3^r} + \cdots + \frac{1}{2016^r}$$

Find the value of

$$\sum_{k=2}^{\infty} f(k)$$

(3) Find the values of the following nested radicals:

i) $\sqrt{5 + \sqrt{5^2 + \sqrt{5^4 + \sqrt{5^8 + \ldots}}}}$

ii) $\sqrt{1 + 2\sqrt{1 + 3\sqrt{1 + 4\sqrt{1 + \cdots}}}}$

(4) Without using a calculator, explain why the following approximation holds:

$$\sqrt{20 + \sqrt{20 + \sqrt{20}}} - \sqrt{20 - \sqrt{20 - \sqrt{20}}} \approx 1$$

(5) Find the length of the leading non-repeating block in the decimal expansion of $\frac{2017}{3\times5^{2016}}$. For example the length of the leading non-repeating block of $\frac{1}{6} = 0.1\overline{6}$ is 1.

(6) Simplify:

i) $C_n^0 + 2C_n^1 + 4C_n^2 + \cdots + 2^n C_n^n$

ii) $C_n^1 + 2C_n^2 + 3C_n^3 + \cdots + nC_n^2$

iii) $C_n^1 + \frac{1}{2}C_n^2 + \frac{1}{3}C_n^3 + \cdots + \frac{1}{n}C_n^n$

iv) $(C_n^0)^2 + (C_n^1)^2 + (C_n^2)^2 \cdots + (C_n^n)^2$

(7) Simplify:

i) $\sin\theta + 2\sin 2\theta + 3\sin 3\theta \cdots + n\sin n\theta$

ii) $\sin\theta + \frac{1}{2}\cdot\sin 2\theta + \frac{1}{4}\cdot\sin 3\theta + \cdots$

iii) $\sin\theta + \frac{1}{2}\sin 2\theta + \frac{1}{4}\sin 3\theta + \cdots$

(8) Without using a calculator, find the value of $\cos\frac{\pi}{13} + \cos\frac{3\pi}{13} + \frac{9\pi}{13}$.