Chapter 1

Introduction

1.1 Contents

This book is not the 2^{nd} edition of the first *Counting* book. Instead, it contains topics which are built upon the contents discussed in the first book. Generally speaking, this book targets AIME, national and international level competitions. Some techniques are quite advanced.

Bijection is a generalization of the symmetry method which is discussed in the first book. While symmetry usually refers to two symmetric parts of the same nature, bijection utilizes one to one mapping among any two sets even if they appear to be completely unrelated. In fact, solutions to some challenging problems discussed in the first book, such as those in the modeling section, essentially are applications of bijection.

The recursion method solves a counting problem by first establishing a recursive relation and then solving this recursion to get the final answer. For example, the Hanoi Tower is a classic problems which can be best tackled using the recursion method. To solve a recursive relation is usually the same as solving a linear regression sequence. The latter is discussed in the book Competition Algebra.

Problems which are related to the integer solution pattern have appeared more and more frequently in recent years. Though necessary solving techniques are covered in the first book, this pattern deserves a more thorough discussion. Counting integer solutions also serves as an introduction to the more powerful generating function method.

Combinatorial identity is an important building block to solve some challenging counting problems. Some of these identities are well known basic relations, and others are not. It is difficult and unnecessary to remember all of these identities. However, it is essential to master core relations and corresponding problem solving techniques such as the special value method, coefficient method, etc.

Generating function will be discussed at the end. It is an advanced general purpose technique that can solve a wide range of math problems, including counting related ones. Usually, generating functions are not required in entry level competitions. However, it is beneficial for those students who aim to excel in national or even international level math contests.

1.2 Notation

In this book, $\binom{n}{k}$ is used instead of C_n^k. These two notations are interchangeable. However, the former style is more convenient when the multinomial theorem is used.