# Contents

**1 Introduction** **1**
1.1 Contents . . . . . 1
1.2 Notation . . . . . 2

**2 Bijection** **3**
2.1 Bijection Explained . . . . . 3
2.2 Examples . . . . . 4
2.3 Practice . . . . . 8

**3 Recursion** **15**
3.1 Introduction . . . . . 15
3.2 More Examples . . . . . 16
3.3 Practice . . . . . 21

**4 Integer Solution and Balls-in-Boxes** **25**
4.1 The Integer Solution Model . . . . . 25
4.2 Count Integer Solutions . . . . . 28
4.3 Other Situations . . . . . 32
4.4 The Balls-in-Boxes Model . . . . . 34
4.5 Practice . . . . . 36

**5 Combinatorial Identities** **41**
5.1 Binomial / Multinomial Theorem . . . . . 41
5.2 The Counting Method . . . . . 42
5.3 The Coefficient Method . . . . . 45
5.4 Pascal Triangle . . . . . 46
5.5 Modular Properties . . . . . 48
5.6 The Special Value Method . . . . . 49
5.7 Hockey Stick Identity . . . . . 53
5.8 Change the Index . . . . . 56
5.9 Generalized Binomial Theorem . . . . . 58
5.10 Practice . . . . . 61

**6 Generating Function** **65**

6.1 Introducing Generating Function . . . . . 65
6.2 Generating Function Properties . . . . . 67
6.3 Useful Conclusions and Techniques . . . . . 68
6.4 Integer Solution Generalization . . . . . 73
6.5 More Examples . . . . . 75
6.6 Practice . . . . . 77

**Appendices** **81**

**A Solutions** **83**
A.1 *Chapter 1* . . . . . 84
A.2 *Chapter 2* . . . . . 85
A.3 *Chapter 3* . . . . . 101
A.4 *Chapter 4* . . . . . 111
A.5 *Chapter 5* . . . . . 121
A.6 *Chapter 6* . . . . . 128