

# Contents

## Preface

<b>1</b>	<b>Introduction</b>	<b>1</b>
1.1	Indeterminate Equation Explained . . . . .	1
1.2	Conventions . . . . .	3
1.3	Practice . . . . .	3
<b>2</b>	<b>The Factorization Method</b>	<b>5</b>
2.1	Solve Equation: $x + xy + y = n$ . . . . .	5
2.2	Solve Equation: $\frac{1}{x} + \frac{1}{y} = \frac{1}{n}$ . . . . .	8
2.3	Difference of Squares . . . . .	10
2.4	More Examples . . . . .	12
2.5	Practice . . . . .	14
<b>3</b>	<b>The Inequality Method</b>	<b>17</b>
3.1	Solve Equation $\frac{1}{x} + \frac{1}{y} = \frac{m}{n}$ . . . . .	17
3.2	Symmetrical Equations . . . . .	18
3.3	Solve Equation: $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{1}{n}$ . . . . .	21
3.4	Sum of Squares . . . . .	23
3.5	The Squeeze Technique . . . . .	25
3.6	Practice . . . . .	28
<b>4</b>	<b>The Quadratic Method</b>	<b>31</b>
4.1	The Principle . . . . .	31
4.2	Examples . . . . .	32
4.3	Practice . . . . .	35
<b>5</b>	<b>The Euclidean Method</b>	<b>37</b>
5.1	Solve Equation: $ax + by = 1$ . . . . .	37
5.2	Bézout's Identity . . . . .	38
5.3	Euclidean Method . . . . .	40
5.4	The MOD Equation Method . . . . .	41
5.5	Practice . . . . .	42

<b>6</b>	<b>General Solution</b>	<b>45</b>
6.1	Special v.s. General Solution . . . . .	45
6.2	General Solution to $ax + by = 1$ . . . . .	47
6.3	Solving $ax + by = c$ . . . . .	49
6.4	Multi-variable Equations . . . . .	53
6.5	Practice . . . . .	56
<b>7</b>	<b>Pythagorean Triplets</b>	<b>59</b>
7.1	Pythagorean Triplets Defined . . . . .	59
7.2	Pythagorean Triplet Formula . . . . .	61
7.3	More Examples . . . . .	63
7.4	Fermat's Last Theorem . . . . .	65
7.5	Practice . . . . .	66
<b>8</b>	<b>The Infinite Descent Method</b>	<b>69</b>
8.1	The Principle . . . . .	69
8.2	Examples . . . . .	71
8.3	Practice . . . . .	73
<b>9</b>	<b>Pell's Equation</b>	<b>75</b>
9.1	Introduction . . . . .	75
9.2	Trivial Cases . . . . .	76
9.3	Solutions to Pell's Equation . . . . .	77
9.3.1	Solve Equation $x^2 - dy^2 = 1$ . . . . .	79
9.3.2	Solve Equation $x^2 - dy^2 = -1$ . . . . .	83
9.4	Alternative Recurrence Relationship . . . . .	85
9.5	More Examples . . . . .	86
9.6	Practice . . . . .	90
<b>10</b>	<b>The MOD Method</b>	<b>93</b>
10.1	When Consider Using MOD . . . . .	93
10.2	Solve Exponential Equations . . . . .	94
10.3	Practice . . . . .	99
	<b>Appendices</b>	<b>101</b>
<b>A</b>	<b>Quick Introduction to Modular Arithmetic</b>	<b>103</b>
A.1	Modular Arithmetic Defined . . . . .	103
A.2	Modular Arithmetic Properties . . . . .	104
A.3	Some Useful Conclusions . . . . .	107

---

<b>B Pell's Equation: Table of Fundamental Solutions</b>	<b>113</b>
<b>C Solutions</b>	<b>115</b>
C.1 Introduction . . . . .	116
C.2 The Factorization Method . . . . .	118
C.3 The Inequality Method . . . . .	125
C.4 The Quadratic Method . . . . .	133
C.5 The Euclidean Method . . . . .	137
C.6 General Solution . . . . .	139
C.7 Pythagorean Triplets . . . . .	146
C.8 The Infinite Descent Method . . . . .	150
C.9 Pell's Equation . . . . .	155
C.10 The MOD Method . . . . .	159