2015-12-10

Counting Assessment

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1. Seven distinct pieces of candy are to be distributed among three bags. The red bag and the blue bag must each receive at least one piece of candy; the white bag may remain empty. How many arrangements are possible?

   (A) 1930 **(B)** 1931 **(C)** 1932 **(D)** 1933 **(E)** 1934

   (01531: 2010 AMC 10 B Q22)

2. In the BIG N, a middle school football conference, each team plays every other team exactly once. If a total of 21 conference games were played during the 2012 season, how many teams were members of the BIG N conference?

   (A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

   (01188: 2012 AMC 08 Q14)

3. A parking lot has 16 spaces in a row. Twelve cars arrive, each of which requires one parking space, and their drivers chose spaces at random from among the available spaces. Auntie Em then arrives in her SUV, which requires 2 adjacent spaces. What is the probability that she is able to park?

   (A) $\frac{11}{20}$ **(B)** $\frac{4}{7}$ **(C)** $\frac{81}{140}$ **(D)** $\frac{3}{5}$ **(E)** $\frac{17}{28}$

   (00790: 2008 AMC 12 B Q22)

4. On the trip home from the meeting where this AMC10 was constructed, the Contest Chair noted that his airport parking receipt had digits of the form $bbcac$, where $0 \le a < b < c \le 9$, and b was the average of a and c. How many different five-digit numbers satisfy all these properties?

   (A) 12 **(B)** 16 **(C)** 18 **(D)** 20 **(E)** 25

   (01667: 2007 AMC 10 B Q8)

5. Find the number of rational numbers r, $0 < r < 1$, such that when r is written as a fraction in lowest terms, the numerator and the denominator have a sum of 1000.

   (00085: 2014 AIME I Q3)

6. In a bag of marbles, $\frac{3}{5}$ of the marbles are blue and the rest are red. If the number of red marbles is doubled and the number of blue marbles stays the same, what fraction of the marbles will be red?

   (A) $\frac{2}{5}$ **(B)** $\frac{3}{7}$ **(C)** $\frac{4}{7}$ **(D)** $\frac{3}{5}$ **(E)** $\frac{4}{5}$

   (00548: 2012 AMC 12 A Q4)

7. A box contains 2 red marbles, 2 green marbles, and 2 yellow marbles. Carol takes 2 marbles from the box at random; then Claudia takes 2 of the remaining marbles at random; and then Cheryl takes the last 2 marbles. What is the probability that Cheryl gets 2 marbles of the same color?

   (A) $\frac{1}{10}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{1}{5}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$

   (00360: 2015 AMC 12 A Q9)

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8. How many 3-digit positive integers have digits whose product equals 24?

   (A) 12 **(B)** 15 **(C)** 18 **(D)** 21 **(E)** 24

   (01060: 2009 AMC 08 Q17)

9. For how many two-element subsets a, b of the set $1, 2, 3, \cdots, 36$ is the product of ab a perfect square?

   (01931: 2012 MathCounts State SPRINT Q29)

10. A magazine printed photos of three celebrities along with three photos of the celebrities as babies. The baby pictures did not identify the celebrities. readers were asked to match each celebrity with the correct baby pictures. What is the probability that a reader guessing at random will match all three correctly?

    (A) $\frac{1}{9}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$

    (00335: 2014 AMC 08 Q12)

11. Bernardo randomly picks 3 distinct numbers from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and arranges them in descending order to form a 3-digit number. Silvia randomly picks 3 distinct numbers from the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$ and also arranges them in descending order to form a 3-digit number. What is the probability that Bernardo's number is larger than Silvia's number?

    (A) $\frac{47}{72}$ **(B)** $\frac{37}{56}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{49}{72}$ **(E)** $\frac{39}{56}$

    (01502: 2010 AMC 10 A Q18)

12. Four boys and four girls line up in a random order. What is the probability that both the first and last person in line is a girl?

    (02000: 2015 Competitions Exeter Speed Q10)

13. Three fair six-sided dice are rolled. What is the probability that the values shown on two of the dice sum to the value shown on the remaining die?

    (A) $\dfrac{1}{6}$ **(B)** $\dfrac{13}{72}$ **(C)** $\dfrac{7}{36}$ **(D)** $\dfrac{5}{24}$ **(E)** $\dfrac{2}{9}$

    (01296: 2014 AMC 10 A Q17)

14. Central High School is competing against Northern High School in a backgammon match. Each school has three players, and the contest rules require that each player play two games against each of the other school's players. The match takes place in six rounds, with three games played simultaneously in each round. In how many different ways can the match be scheduled?

    (A) 540 **(B)** 600 **(C)** 720 **(D)** 810 **(E)** 900

    (01353: 2013 AMC 10 A Q24)

15. The nine delegates to the Economic Cooperation Conference include 2 officials from Mexico, 3 officials from Canada, and 4 officials from the United States. During the opening session,

three of the delegates fall asleep. Assuming that the three sleepers were determined randomly, the probability that exactly two of the sleepers are from the same country is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

(00054: 2015 AIME I Q2)

16. Everyday at school, Jo climbs a flight of 6 stairs. Joe can take the stairs 1, 2, or 3 at a time. For example, Jo could climb 3, then 1, then 2. In how many ways can Jo climb the stairs?

    (A) 13 **(B)** 18 **(C)** 20 **(D)** 22 **(E)** 24

    (01150: 2010 AMC 08 Q25)

17. A word is an ordered, non-empty sequence of letters, such as word or wrod. How many distinct words can be made from a subset of the letters *c, o, m, b, o*, where each letter in the list is used no more than the number of times it appears?

    (02610: 2015 Competitions PUMAC Combinatorics A Q1)

18. How many even three-digit integers have the property that their digits, read left to right, are in strictly increasing order?

    (A) 21 (B) 34 (C) 51 (D) 72 (E) 150

    (00877: 2006 AMC 12 B Q9)

19. A dessert chef prepares the dessert for every day of a week starting with Sunday. The dessert each day is either cake, pie, ice cream, or pudding. The same dessert may not be served two days in a row. There must be cake on Friday because of a birthday. How many different dessert menus for the week are possible?

    (A) 729 **(B)** 972 **(C)** 1024 **(D)** 2187 **(E)** 2304

    (01415: 2012 AMC 10 B Q11)

20. At a certain university, the division of mathematical sciences consists of the departments of mathematics, statistics, and computer science. There are two male and two female professors in each department. A committee of six professors is to contain three men and three women and must also contain two professors from each of the three departments. Find the number of possible committees that can be formed subject to these requirements.

    (00227: 2012 AIME II Q3)

21. Let S be a square of side length 1. Two points are chosen independently at random on the sides of S. The probability that the straight-line distance between the points is at least $\dfrac{1}{2}$ is $\dfrac{a-b\pi}{c}$, where a, b, and c are positive integers with $\gcd(a,b,c)=1$. What is $a+b+c$?

    (A) 59 **(B)** 60 **(C)** 61 **(D)** 62 **(E)** 63

    (01249: 2015 AMC 10 A Q25)

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22. A coin is altered so that the probability that it lands on heads is less than $\frac{1}{2}$ and when the coin is flipped four times, the probability of an equal number of heads and tails is $\frac{1}{6}$. What is the probability that the coin lands on heads?

    (A) $\frac{\sqrt{15}-3}{6}$ **(B)** $\frac{6-\sqrt{6\sqrt{6}+2}}{12}$ **(C)** $\frac{\sqrt{2}-1}{2}$ **(D)** $\frac{3-\sqrt{3}}{6}$ **(E)** $\frac{\sqrt{3}-1}{2}$

    (00658: 2010 AMC 12 A Q15)

23. Ten people form a line, among which two are Chinese and two are Americans. The probability that both Chinese will stand in front of both Americans (not necessarily immediately in the front) can be expressed as $\frac{p}{q}$ where p and q are two relatively prime positive integers. Find $p+q$

    (00003)

24. A traffic light runs repeatedly through the following cycle: green for 30 seconds, then yellow for 3 seconds, and then red for 30 seconds. Leah picks a random three-second time interval to watch the light. What is the probability that the color changes while she is watching?

    (A)$\frac{1}{63}$ (B)$\frac{1}{21}$ (C)$\frac{1}{10}$ (D)$\frac{1}{7}$ (E)$\frac{1}{3}$

    (00831: 2007 AMC 12 B Q13)

25. An unfair coin lands on heads with a probability of $\frac{1}{4}$. When tossed n times, the probability of exactly two heads is the same as the probability of exactly three heads. What is the value of n ?

    (A) 5 **(B)** 8 **(C)** 10 **(D)** 11 **(E)** 13

    (00395: 2015 AMC 12 B Q17)