

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>Limit</b>	<b>3</b>
2.1	Limit Defined . . . . .	3
2.2	Limit Computation . . . . .	6
2.3	The Sandwich Theorem . . . . .	8
2.4	Bounded Monotonic Function . . . . .	11
2.5	Left, Right Limit and Continuity . . . . .	13
2.6	Examples and Applications . . . . .	16
2.6.1	Finding Asymptotes . . . . .	16
2.6.2	Sum Infinite Series . . . . .	18
2.6.3	Compute Area Under A Curve . . . . .	19
2.6.4	Continuously Compounded Interest . . . . .	20
2.7	Practice . . . . .	22
<b>3</b>	<b>Derivative</b>	<b>25</b>
3.1	Derivative Defined . . . . .	25
3.2	Differentiability vs Continuity . . . . .	27
3.3	Compute Derivative (I) . . . . .	28
3.4	Derivative's Properties . . . . .	30
3.5	Derivative of $e^x$ . . . . .	32
3.6	Notation Demystification . . . . .	33
3.7	Implicit Derivative . . . . .	34
3.8	Differentiate Parametric Function . . . . .	35
3.9	Compute Derivative (II) . . . . .	36
3.10	Inverse Function Rule . . . . .	37
3.11	Derivatives of Trigonometric Functions . . . . .	39
3.12	The Chain Rule . . . . .	40
3.13	The Quotient Rule . . . . .	43
3.14	Table of Common Derivatives . . . . .	44
3.15	Additional Techniques . . . . .	45
3.15.1	Differentiate $x^x$ . . . . .	45
3.15.2	Product Rule For Higher Order . . . . .	46
3.16	Concavity . . . . .	47

## CONTENTS

---

3.17	Partial Derivative . . . . .	49
3.18	Examples and Applications . . . . .	51
3.18.1	Determine Minimum and Maximum . . . . .	51
3.18.2	Determine Inflection Points . . . . .	55
3.18.3	The L'Hôpital Rule . . . . .	55
3.18.4	Root Finding Algorithm . . . . .	57
3.18.5	Regression and Machine Learning . . . . .	60
3.19	Practice . . . . .	62
<b>4</b>	<b>Integral</b>	<b>67</b>
4.1	Rectangular Approximation Model . . . . .	67
4.2	Riemann Sum and Integral . . . . .	68
4.3	Fundamental Theorem of Calculus . . . . .	70
4.4	The Substitution Method . . . . .	74
4.5	Trigonometric Substitution . . . . .	76
4.6	Integration By Parts . . . . .	78
4.7	Additional Techniques . . . . .	80
4.7.1	Watch Out Absolute Value . . . . .	81
4.7.2	The Symmetry Method . . . . .	82
4.7.3	The Symmetry Method (II) . . . . .	83
4.7.4	Special Pattern . . . . .	84
4.7.5	Partial Fraction Decomposition . . . . .	85
4.7.6	Completing the Square . . . . .	87
4.7.7	The Construction Method . . . . .	88
4.7.8	Recursion . . . . .	91
4.8	Improper Integral . . . . .	92
4.9	Differential Equation . . . . .	95
4.9.1	Separable Differential Equation . . . . .	95
4.9.2	Integrating Factor . . . . .	96
4.9.3	Homogeneous Equation . . . . .	97
4.10	Examples and Applications . . . . .	98
4.10.1	Compute Arc Length . . . . .	99
4.10.2	Compute Area Using Polar Coordinates . . . . .	101
4.10.3	Compute Volume . . . . .	102
4.10.4	Compute Surface Area . . . . .	104
4.10.5	Determine Center of Mass . . . . .	106
4.10.6	Derivative and Integral in Physics . . . . .	110
4.10.7	Differential Equation in Physics . . . . .	111
4.11	Practice . . . . .	113
<b>5</b>	<b>Infinite Series</b>	<b>119</b>
5.1	Convergence . . . . .	119