

Contents

1	Introduction	1
2	Go Simple and Go Extreme	3
2.1	The Golden Principle	3
2.2	The Special Value Technique	5
2.3	Solve Functional Equation	7
2.4	Additional Application	10
2.5	Practice	12
3	Induction and Recursion	17
3.1	Base Case and Inductive Step	17
3.2	Variations	21
3.3	Recursion	24
3.4	Practice	27
4	Proof by Contradiction	31
4.1	Attack from the Opposite	31
4.2	More Examples	32
4.3	Practice	35
5	Pigeonhole Principle	37
5.1	Pigeonhole Principle Explained	37
5.2	More Examples	39
5.3	Practice	43
6	The Coloring Method	47
6.1	Examples	47
6.2	Practice	51
7	Two-State Problem	55
7.1	Introduction	55
7.2	Odd Even Parity	55
7.3	The ± 1 Model	58
7.4	Invariant	59

CONTENTS

7.5	Practice	60
8	Symmetry	65
8.1	Beyond Geometric Symmetry	65
8.2	Solving Algebra Problems	66
8.2.1	Symmetric Polynomial Factorization	66
8.2.2	Symmetric Equations and Inequalities	68
8.2.3	Symmetric Indeterminate Equation	70
8.3	Solving Counting Problems	72
8.4	Practice	73
	Appendices	77
A	Solutions	79
A.1	Introduction	79
A.2	Go Simple and Go Extreme	80
A.3	Induction and Recursion	93
A.4	Proof by Contradiction	103
A.5	Pigeonhole Principle	108
A.6	The Coloring Method	118
A.7	Two-State Problem	127
A.8	Symmetry	137