

Counting

# Counting in Number Theory



Learn how to solve this *type* of problems, not just this problem.

---



*Tip: Always write down intermediate steps.*

(1) How many positive divisors does 20 have?

(Ref: 2481)

(2) How many integer solutions does the equation  $(x + 1)(y + 1) = 25$  have?

(Ref: 2483)

(3) Find the number of different rectangles that satisfy the following conditions:

(a) its area is 2015

(b) the lengths of all its sides are integers

(Ref: 2482)

(4) How many positive integers, not exceeding 2015, are relatively prime to 2015?

(Ref: 2506)

(5) Let  $p$  be a prime number, computer  $\phi(p)$ .

(Ref: 2507)

(6) Let  $p$  be a prime number and  $n$  be a positive integer. Show that  $\phi(p^n) = p^n - p^{n-1}$ .

(Ref: 2508)

(7) Compute  $\phi(5)$ ,  $\phi(6)$ , and  $\phi(30)$ . What relationship among these answers have you noticed?

(8) Show that if  $a$  and  $b$  are relatively prime, then  $\phi(a)\phi(b) = \phi(ab)$ .

(Ref: 2509)

(9) How many fraction numbers between 0 and 1 are there whose denominator is 1001 when written in its simplest form?

(Ref 2723)