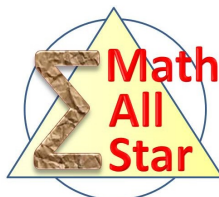

Trigonometry

Basics Reinforcement



Math for Gifted Students

<http://www.mathallstar.org>




Basics Reinforcement



Instructions

- Write down and submit intermediate steps along with your final answer.
- If the final result is too complex to compute, give the expression. e.g. C_{100}^{50} is acceptable.
- Problems are not necessarily ordered based on their difficulty levels.
- Always ask yourself what makes this problem a good practice?
- Read through the reference solution even if you can solve the problem for additional information which may help you to solve this type of problems.

Legends

-  *Tips, additional information etc*
-  *Important theorem, conclusion to remember.*
-  *Addition questions for further study.*

My Comments and Notes

Basics Reinforcement



Practice 1

Convert between radian and degree measures:

- | | | |
|------------------|------------------------|------------------------|
| (i) 30° | (v) 120° | (viii) $\frac{\pi}{6}$ |
| (ii) 45° | (vi) 270° | (ix) $-\frac{3\pi}{5}$ |
| (iii) 60° | (vii) $-\frac{\pi}{4}$ | (x) 2π |
| (iv) 90° | | |

Practice 2

Complete the following table:

	0°	30°	45°	60°	90°	120°	150°	270°	360°	540°
sin										
cos										
tan										

Practice 3

Which of the following equations always hold?

- | | |
|------------------------------------------------|---------------------------------------------------------------|
| (i) $\sin^2 \theta + \cos^2 \theta = 1$ | (v) $\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$ |
| (ii) $\tan \theta = \cos \theta / \sin \theta$ | (vi) $\sin\left(\frac{\pi}{2} + \theta\right) = -\cos \theta$ |
| (iii) $\sin(-\theta) = \sin \theta$ | (vii) $1 + \tan^2 \theta = \frac{1}{\cos^2 \theta}$ |
| (iv) $\cos(\pi - \theta) = \cos \theta$ | |

Practice 4

What are the ranges of the *sin*, *cos*, and *tan* function, respectively?

Basics Reinforcement

**Practice 5**

When θ increases from 0 to $\frac{\pi}{2}$, determine whether each statement below is true or not:

- (i) the value of $\sin \theta$ increases
- (ii) the value of $\cos \theta$ increases
- (iii) the value of $\tan \theta$ increases

Practice 6

When $\frac{\pi}{4} < \theta < \frac{\pi}{2}$, which of the following statement holds?

- (i) $\sin \theta > \cos \theta > \tan \theta$
- (ii) $\cos \theta > \tan \theta > \sin \theta$
- (iii) $\tan \theta > \sin \theta > \cos \theta$
- (iv) $\sin \theta > \tan \theta > \cos \theta$

Practice 7

Let x be a real number and $0 \leq x \leq \frac{\pi}{2}$, explain why the following inequality holds and when the equality sign holds:

$$\sin x \leq x \leq \tan x$$

Practice 8

Find all angles $\theta \in [0, 2\pi)$ such that $\sin \theta = \frac{1}{2}$. Express your answer in radian.

Trigonometry

Basics Reinforcement



Practice 9

Find all angles θ such that $\sin \theta = \frac{1}{2}$. Express your answer in radian.

Practice 10

Find all angles θ such that $\sin \theta \leq \frac{1}{2}$. Express your answer in radian.



Answer Keys

Practice 1:

- | | | |
|----------------------------------|------------------------------------|------------------------------------|
| (i) $30^\circ = \frac{\pi}{6}$ | (v) $120^\circ = \frac{2\pi}{3}$ | (viii) $\frac{\pi}{6} = 30^\circ$ |
| (ii) $45^\circ = \frac{\pi}{4}$ | (vi) $270^\circ = \frac{3\pi}{2}$ | (ix) $-\frac{3\pi}{5} = 108^\circ$ |
| (iii) $60^\circ = \frac{\pi}{3}$ | (vii) $-\frac{\pi}{4} = -45^\circ$ | (x) $2\pi = 360^\circ$ |
| (iv) $90^\circ = \frac{\pi}{2}$ | | |

Practice 2:

	0°	30°	45°	60°	90°	120°	150°	270°	360°	540°
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	-1	0	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	0	1	-1
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$+\infty$	$-\sqrt{3}$	$-\frac{\sqrt{3}}{3}$	$-\infty$	0	0

Practice 3:

- | | |
|-------------|------------|
| (i) TRUE | (v) TRUE |
| (ii) FALSE | (vi) TRUE |
| (iii) FALSE | (vii) TRUE |
| (iv) FALSE | |

Trigonometry

Basics Reinforcement



Practice 4: $[-1, 1]$, $[-1, 1]$, $(-\infty, +\infty)$

Practice 5:

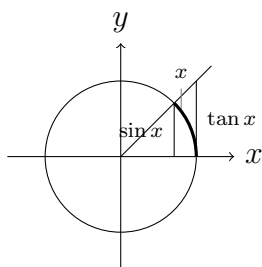
(i) True

(ii) False

(iii) True

Practice 6: (iii)

Practice 7:



The equality holds when $x = 0$.

Practice 8: $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

Practice 9: $\theta = 2k\pi + \frac{\pi}{6}, 2k\pi + \frac{5\pi}{6}$ where k is an integer, or

$\theta = k\pi + (-1)^k \cdot \frac{\pi}{6}$ where k is an integer.

Practice 10:

$\theta \in [2k\pi, (2k + \frac{1}{6})\pi] \cup [2k\pi + \frac{5\pi}{6}, (2k + 2)\pi]$, where $k \in \mathbb{Z}$, or

$\theta \in [(2k - \frac{7}{6})\pi, (2k + \frac{1}{6})\pi]$, where $k \in \mathbb{Z}$

