Indeterminate Equation

The Infinite Descending Method

Learn how to solve this *type* of problems, not just this problem.

1. Describe the principle idea of the infinite descending method.

2. Solve in positive integers
$$x^3 + y^3 + z^3 = 3xyz$$
(Ref Ref 2353)

3. Show that the equation $x^4 + y^4 = z^2$ is not solvable in integers if $xyz \neq 0$.
(Ref Ref 2319)

4. Show that the equation $x^4 + y^4 = z^4$ is not solvable in positive integer.

5. Show that if the equation $x^n + y^n = z^n$ is not solvable in positive integer for a given positive integer n, then the equation
$$x^{2n} + y^{2n} = z^{2n}$$
is not solvable in positive integers either.
(Ref Ref 2351)

6. Show that the sum and difference of two squares cannot be both squares themselves.
(Ref Ref 2350)

7. Find all primes p for which there exist positive integers x, y, and n such that
$$p^n = x^3 + y^3$$
(Ref Ref 2322: 2000 Hungarian Olympiad)

8. Prove that if positive integer a and b are such that $ab + 1$ divides $a^2 + b^2$. then
$$\frac{a^2 + b^2}{ab + 1}$$
is a square number.
(Ref Ref 2324: 1988 IMO)