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Indeterminate Equation

The Infinite Descending Method



Learn how to solve this *type* of problems, not just this problem.

- 1. Describe the principle idea of the infinite descending method.
- 2. Solve in positive integers

$$x^3 + y^3 + z^3 = 3xyz$$

(Ref Ref 2353)

- 3. Show that the equation $x^4 + y^4 = z^2$ is not solvable in integers if $xyz \neq 0$. (Ref Ref 2319)
- 4. Show that the equation $x^4 + y^4 = z^4$ is not solvable in positive integer.
- 5. Show that if the equation $x^n + y^n = z^n$ is not solvable in positive integer for a given positive integer n, then the equation

$$x^{2n} + y^{2n} = z^{2n}$$

is not solvable in positive integers either. (Ref Ref 2351)

- 6. Show that the sum and difference of two squares cannot be both squares themselves. $_{\rm (Ref\ Ref\ 2350)}$
- 7. Find all primes p for which there exist positive integers x, y, and n such that

$$p^n = x^3 + y^3$$

(Ref Ref 2322: 2000 Hungarian Olympiad)

8. Prove that if positive integer a and b are such that ab + 1 divides $a^2 + b^2$, then

$$\frac{a^2 + b^2}{ab + 1}$$

is a square number. (Ref Ref 2324: 1988 IMO)