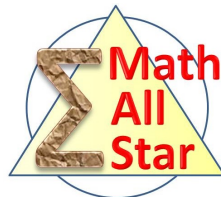


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# Art of Thinking

## Pigeonhole Principle

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*Math for Gifted Students*

<http://www.mathallstar.org>

Art of Thinking

# Pigeonhole Principle



## Instructions

- Write down and submit intermediate steps along with your final answer.
- If the final result is too complex to compute, give the expression. e.g.  $C_{100}^{50}$  is acceptable.
- Problems are not necessarily ordered based on their difficulty levels.
- Always ask yourself what makes this problem a good one to practise?
- Complete the My Record section below before submission.

## My Comments and Notes

Sample

# Pigeonhole Principle



## Practice 1

With 13 randomly selected points inside a square of side length 2, show that there must exist quadrilateral whose vertices are among these 13 points and area is no more than 1.

(Ref 2793)

## Practice 2

Show that among any four randomly selected integers, at least two of them must have a difference which is a multiple of 3.

## Practice 3

Let  $a_1, a_2, a_3, \dots, a_9$  be a random permutation of 1, 2, 3, ..., 9. Prove

$$(a_1 - 1)(a_2 - 2) \cdots (a_9 - 9)$$

is an even number.

(Ref UK Olympiad)

## Practice 4

Prove: randomly select 51 numbers from 1, 2, 3, ..., 100, there must exist two numbers for which one is a multiple of the other.

(Ref 1119)

## Practice 5

Show that it is possible to find an integer whose digits are all 8 and it is a multiple of 2016.

(Ref 2792)

## Practice 6

Prove: any convex pentagon must have three vertices  $A$ ,  $B$ , and  $C$ , such that  $\angle ABC \leq 36^\circ$ .

(Ref 2105)

# Pigeonhole Principle



## Battle Field

Here are some related problems selected from recent competitions:

Problem 1: 2012 MathCounts State Target #5

Problem 2: 2002 AMC10P #15

Sample