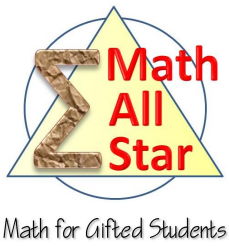
Art of Thinking

Pigeonhole Principle



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Pigeonhole Principle

Instructions

- *Write down and submit intermediate steps along with your final answer.*
- *If the final result is too complex to compute, give the expression. e.g.* C_{100}^{50} *is acceptable.*
- *Problems are not necessarily ordered based on their difficulty levels.*
- *Always ask yourself what makes this problem a good one to practise?*
- *Complete the My Record section below before submission.*

My Comments and Notes

Pigeonhole Principle

Practice 1

With 13 randomly selected points inside a square of side length 2, show that there must exist quadrilateral whose vertice are among these 13 points and area is no more than 1.

(Ref 2793)

Practice 2

Show that among any four randomly selected integers, at least two of them must have a difference which is a multiple of 3.

Practice 3

Let a_1, a_2, a_3, $\cdots$, a_9 be a random permutation of 1, 2, 3, .., 9. Prove

$$(a_1 - 1)(a_2 - 2)\cdots(a_9 - 9)$$

is an even number.

(Ref UK Olympiad)

Practice 4

Prove: randomly select 51 numbers from 1, 2, 3, ..., 100, there must exist two numbers for which one is a multiple of the other.

(Ref 1119)

Practice 5

Show that it is possible to find an integer whose digits are all 8 and it is a multiple of 2016.

(Ref 2792)

Practice 6

Prove: any convex pentagon must have three vertex A, B, and C, such that $\angle ABC \leq 36°$.

(Ref 2105)

Battle Field

Here are some related problems selected from recent comptitions:

Problem 1: 2012 MathCounts State Target #5

Problem 2: 2002 AMC10P #15