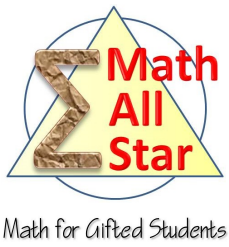
Geometry

Coordinate Basics



http://www.mathallstar.org

Geometry

Coordinate Basics

Instructions

- *Write down and submit intermediate steps along with your final answer.*
- *If the final result is too complex to compute, give the expression. e.g.* C_{100}^{50} *is acceptable.*
- *Problems are not necessarily ordered based on their difficulty levels.*
- *Always ask yourself what makes this problem a good practice?*
- *Read through the reference solution even if you can solve the problem for additional information which may help you to solve this type of problems.*

Legends

Tips, additional information etc

Important theorem, conclusion to remember.

Addition questions for further study.

My Comments and Notes

Coordinate Basics

The emphasis of this practice is to write the required equations or formulas **directly**.

You should remember all the conclusions in this practice.

Coordinate Basics

Practice 1

(Distance) What is the distance between (x_1, y_1) and (x_2, y_2)?

Practice 2

(Middle Point) What is the coordinate of the middle point between (x_1, y_1) and (x_2, y_2)?

Practice 3

(Centroid) Give a triangle whose vertices are $(x_1, y_1), (x_2, y_2)$, and (x_3, y_3), respectively. What is the coordinate of its center of mass (centroid)?

Practice 4

(Incenter) Give a triangle whose vertices are $A(x_a, y_a), B(x_b, y_b)$, and $C(x_c, y_c)$, respectively. If $a = BC, b = CA$, and $c = BA$, what is the coordinate of its incenter?

Practice 5

(Interpolation) Give two points $A(x_a, y_a)$ and $B(x_b, y_b)$, a point C on $\overline{AB}$. If $AC : CB = m : n$ where m and n are two integers, find the coordinate of point C.

Practice 6

(Line by Two Points) If a straight line l passes two distinct points (x_1, y_1) and (x_2, y_2), what is l's equation?

Practice 7

(Line by Intercepts) What is the equation of a straight line if its x-intercept and y-intercept are a and b, respectively?

Coordinate Basics

Practice 8

(Plane by Intercepts) What is the equation of a plane if its x-intercept, y-intercept, and z-intercept are a, b, and c, respectively?

Practice 9

(Point to Line) What is the distance from the point (x_0, y_0) to the line $Ax + By + C = 0$?

Practice 10

(Point to Plane) What is the distance from the point (x_0, y_0, z_0) to the plane $Ax + By + Cz + D = 0$?

Practice 11

(Triangle Area) What is the area of a triangle whose vertices are $(x_1, y_1), (x_2, y_2)$, and (x_3, y_3)?

Practice 12

(Polygon Area) What is the area of a polygon whose vertices are $(x_1, y_1), (x_2, y_2), \cdots$ and (x_n, y_n)?

Practice 13

Given two parallel lines: $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$, find the locus of all the points that are equidistant to these two lines.

Coordinate Basics

Practice 14

(Circle Tangent Line) Let point $P(x_0, y_0)$ be on circle O, find the equation of the straight line which is tangent to O on point P when O is given by:

(i) $x^2 + y^2 = r^2$

(ii) $(x - a)^2 + (y - b)^2 = r^2$

(iii) $x^2 + y^2 + Dx + Ey + F = 0$

Practice 15

(Chord Passing Tangent Points) Let point $P(x_0, y_0)$ be outside the circle $O : x^2 + y^2 = 0$. If PA and PB are two lines that pass P and are tangent to O at A and B, find the equation of line AB.

Practice 16

(Distance to Tangent Points) Let point $P(x_0, y_0)$ be outside the circle O. If PA is a line that passes P and is tangent to O at A, find the distance between P and A when O is given by:

(i) $x^2 + y^2 = r^2$

(ii) $(x - a)^2 + (y - b)^2 = r^2$

(iii) $x^2 + y^2 + Dx + Ey + F = 0$

Reference Solutions

Coordinate Basics

Practice 1

(Distance) What is the distance between (x_1, y_1) and (x_2, y_2)?

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Practice 2

(Middle Point) What is the coordinate of the middle point between (x_1, y_1) and (x_2, y_2)?

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Practice 3

(Centroid) Give a triangle whose vertices are $(x_1, y_1), (x_2, y_2)$, and (x_3, y_3), respectively. What is the coordinate of its center of mass (centroid)?

$$\left(\frac{x_1 + x_2 + x_3}{x}, \frac{y_1 + y_2 + y_3}{3}\right)$$

Practice 4

(Incenter) Give a triangle whose vertices are $A(x_a, y_a), B(x_b, y_b)$, and $C(x_c, y_c)$, respectively. If $a = BC, b = CA$, and $c = BA$, what is the coordinate of its incenter?

$$\left(\frac{ax_a + bx_b + cx_c}{a + b + c}, \frac{ay_a + by_b + cy_c}{a + b + c}\right)$$

Coordinate Basics

Practice 5

(Interpolation) Give two points $A(x_a, y_a)$ and $B(x_b, y_b)$, a point C on $\overline{AB}$. If $AC : CB = m : n$ where m and n are two integers, find the coordinate of point C.

$$\left(\frac{nx_a + mx_b}{m+n}, \frac{ny_a + my_b}{m+n}\right)$$

Practice 6

(Line by Two Points) If a straight line l passes two distinct points (x_1, y_1) and (x_2, y_2), what is l's equation?

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1} \quad or \quad \frac{y - y_2}{x - x_2} = \frac{y_2 - y_1}{x_2 - x_1}$$

Practice 7

(Line by Intercepts) What is the equation of a straight line if its x-intercept and y-intercept are a and b, respectively?

$$\frac{x}{a} + \frac{y}{b} = 1$$

Practice 8

(Plane by Intercepts) What is the equation of a plane if its x-intercept, y-intercept, and z-intercept are a, b, and c, respectively?

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Coordinate Basics

Practice 9

(Point to Line) What is the distance from the point (x_0, y_0) to the line $Ax + By + C = 0$?

$$\frac{| Ax_0 + By_0 + C |}{\sqrt{A^2 + B^2}}$$

Practice 10

(Point to Plane) What is the distance from the point (x_0, y_0, z_0) to the plane $Ax + By + Cz + D = 0$?

$$\frac{| Ax_0 + By_0 + Cz_0 + D |}{\sqrt{A^2 + B^2 + C^2}}$$

Practice 11

(Triangle Area) What is the area of a triangle whose vertices are $(x_1, y_1), (x_2, y_2)$, and (x_3, y_3)?

$$\frac{| (x_1y_2 - y_1x_2) + (x_2y_3 - y_2x_3) + (x_3y_1 - y_3x_1) |}{2}$$

Practice 12

(Polygon Area) What is the area of a polygon whose vertices are $(x_1, y_1), (x_2, y_2), \cdots$ and (x_n, y_n)?

$$\frac{| (x_1y_2 - y_1x_2) + (x_2y_3 - y_2x_3) + \cdots (x_ny_1 - y_nx_1) |}{2}$$

Coordinate Basics

Practice 13

Given two parallel lines: $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$, find the locus of all the points that are equidistant to these two lines.

$$Ax + By + \frac{C_1 + C_2}{2} = 0$$

Practice 14

(Circle Tangent Line) Let point $P(x_0, y_0)$ be on circle O, find the equation of the straight line which is tangent to O on point P when O is given by:

(i) $x^2 + y^2 = r^2$

(ii) $(x - a)^2 + (y - b)^2 = r^2$

(iii) $x^2 + y^2 + Dx + Ey + F = 0$

(i) $x_0x + y_0y = r^2$

(ii) $(x_0 - a)(x - a) + (y_0 - b)(y - b) = r^2$

(iii) $x_0x + y_0y + D \cdot \frac{x_0 + x}{2} + E \cdot \frac{y_0 + y}{2} + F = 0$

Practice 15

(Chord Passing Tangent Points) Let point $P(x_0, y_0)$ be outside the circle $O : x^2 + y^2 = 0$. If PA and PB are two lines that pass P and are tangent to O at A and B, find the equation of line AB.

$$x_0x + y_0y = r^2$$

Coordinate Basics

Practice 16

(Distance to Tangent Points) Let point $P(x_0, y_0)$ be outside the circle O. If PA is a line that passes P and is tangent to O at A, find the distance between P and A when O is given by:

(i) $x^2 + y^2 = r^2$

(ii) $(x - a)^2 + (y - b)^2 = r^2$

(iii) $x^2 + y^2 + Dx + Ey + F = 0$

(i) $\sqrt{x_0^2 + y_0^2 - r^2}$

(ii) $\sqrt{(x_0 - a)^2 + (y_0 - b)^2 - r^2}$

(iii) $\sqrt{x_0^2 + y_0^2 + Dx_0 + Ey_0 + F}$

Tip: How can you remember these three formulas?

Battle Field

Selective problems from recent comptitions:

Problem 1: 2016 AMC10B #20 (Ref 2925)

Problem 2: 2015 AIME II #9 (Ref 76)

Problem 3: 2015 AIME I #4 (Ref 56)

Problem 4: 2013 AMC12A #13 (Ref 489)

Problem 5: 2013 MathCounts State Target #6 (Ref 1841)

Problem 6: 2012 AMC10B #23 (Ref 1427)

Problem 7: 2012 MathCounts Chapter Target #3 (Ref 1975)

Problem 8: 2012 MathCounts Chapter Team #2 (Ref 1982)