Counting

# Balls in Boxes

Learn how to solve this *type* of problems, not just this problem.

💡 *Tip: Always write down intermediate steps.*

(1) Given an equation $x_1 + x_2 + \cdots + x_k = n$, how to count its

    (a) positive integer solutions

    (b) non-negative integer solutions

(2) Explain why we cannot model distinct balls as integers.

(3) Consider the equation $x_1 + x_2 + x_3 + \cdots + x_6 = 20$. Answer the following questions:

    (a) How many positive integer solution does it have?

    (b) How many non-negative integer solutions does it have?

    (c) How many integer solutions are there, if $x_1 \geq 0$ and $x_2, x_3, \cdots x_6 > 0$?

(4) Team MAS won a total of 10 gold medals in a 6-day tournament. It won at least one gold medal every day. How many different possibilities are there to count the number of gold medals won each day?

(Ref: 2525)

(5) Joe goes to a supermarket to buy 10 cakes. There are 6 different types of cakes, and each type has a sufficient quantity. How many different combinations of cakes can Joe have?

(Ref: 2527)

(6) How many different ways to expression 10 as sum of some positive integers? The two expressions: $10 = 4 + 6$ counts as a different result from $10 = 6 + 4$.

(7) A parking lot has 16 spaces in a row. Twelve cars arrive, each of which requires one parking space, and their drivers chose spaces at random from among the available spaces. Auntie Em then arrives in her SUV, which requires 2 adjacent spaces. What is the probability that she is able to park?

(Ref 790: 2008 AMC12B #22)

(8) Write down the expression, no need to compute the final result:

How many different ways are there to put 10 different balls into 5 different boxes such that no box is empty or contains more than 4 balls.

(Ref 2658)