Indeterminate Equation

The Inequality (Squeeze) Method

Learn how to solve this *type* of problems, not just this problem.

1. Solve this equation in integers: $y^2 = x^2 + x + 1$.
2. Solve in integers the equation $y^2 = x^4 + x^3 + x^2 + x + 1$.
   (Ref Ref 2081)
3. Solve in integers the equation
   $$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{3}{5}$$
   (Ref Ref 2084: Romania Olympiad)
4. Solve in positive integers the equation
   $$3(xy + yz + zx) = 4xyz$$
   (Ref Ref 2243: Putname)
5. A rectangular box measures $a \times b \times c$, where a, b, and c are integers and $1 \le a \le b \le c$. The volume and the surface area of the box are numerically equal. How many ordered triples (a, b, c) are possible?
   (Ref Ref 401: 2015 AMC12B #23)
6. Solve in integers the equation
   $$(x + y)^2 = x^3 + y^3$$
   (Ref Ref 2359)
7. How many ordered triples of integers (a, b, c), with $a \ge 2$, $b \ge 1$, and $c \ge 0$, satisfy both $\log_a b = c^{2005}$ and $a + b + c = 2005$?
   (Ref Ref 914: 2005 AMC12A #21)
8. Find all integers a, b, c with $1 < a < b < c$ such that the number $(a-1)(b-1)(c-1)$ is a divisor of $abc - 1$.
   (Ref Ref 2240: 1992 IMO)
9. Find all positive integers n and k_i $(1 \le i \le n)$ such that
   $$k_1 + k_2 + \cdots + k_n = 5n - 4$$
   and
   $$\frac{1}{k_1} + \frac{1}{k_2} + \cdots + \frac{1}{k_n} = 1$$
   (Ref Ref 2242: Putnam)
10. Solve in positive integers $(1 + \frac{1}{x})(1 + \frac{1}{y})(1 + \frac{1}{z}) = 2$
    (Ref Ref 2241: UK Olympiad)